International Journal of Engineering Innovation & Research
Volume 2, Issue 1, ISSN: 2277 – 5668

Fixed Point Theorem in Fuzzy Metric Space by using Compatibility of Type α

M. S. Chauhan
Asst. Professor
Govt. Mandideep Collge Raisen (M.P.)

Manoj Kumar Khanduja
Lecturer SOC. and E.
IPS Academy Indore (M.P.)
manojkhanduja3@gmail.com

Bharat Singh
Reader SOC. and E.
IPS Academy Indore (M.P.)
bharat_singhips@yahoo.com

***Abstract* – In this paper we give a fixed point theorem on fuzzy metric space with compatibility of type (∝) Our result extends and generalize the result of Singh and Chauhan [8].**

***Keywords* – Fuzzy Metric Space, Type (∝) Mappings, Type (β) Mappings.**

I. Introduction

Zadeh [11] introduced the concept of fuzzy sets in 1965 and in the next decade Kramosil and Michalek [12] introduced the concept of fuzzy metric spaces in 1975, which opened an avenue for further development of analysis in such spaces.Vasuki [13] investigated same fixed point theorem s in fuzzy metric spaces for R-weakly commuting mappings and pant [14] introduced the notion of reciprocal continuity of mappings in metric spces.Balasubramaniam et al and S. Muralishankar,R.P. Pant [15] proved the poen problem of Rhodes [16] on existence of a contractive definition. Recently, Cho et al [2] initiated the concept of compatible maps of type (β) in fuzzy metric spaces by giving interesting relationship of these type of mapping with compatible and compatible of type (□) mappings.

II. Preliminaries

Definition 2.1. A binary operation 8 * : [0, 1] × [0, 1] → [0, 1] is called a t –norm if ([0, 1], *) is an abelian topological monoid with unit 1 such that a * b ≤ c * d whenever a ≤ c and b ≤ d for a, b, c, d □ [0, 1].
Examples of t-norms are a *b = ab and a * b = min{a, b}
Definition 2.2. ([9]) The 3-tuple (X,M, *) is called a fuzzy metric space , if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in X2 × [0,∞) satisfying the following conditions: for all x, y, z □ X and s, t > 0.
(F.M-1) M(x, y, 0) = 0,
(F.M-2) M(x, y, t) = 1 for all t > 0 if and only if x = y,
(F.M-3) M(x, y, t) = M(y, x, t),
(F.M-4) M(x, y, t) *M(y, z, s) ≤ M(x, z, t + s),
(F.M-5) M(x, y, ·) : [0,∞) × [0, 1] is left continuous,
(F.M-6) M(x, y, t) = 1.

Note that M(x, y, t) can be considered as the degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) =1 for all t > 0.
Example 2.1. ([4]) Let (X, d) be a metric space. Define a * b =min{a, b} and $M(x,y,t) = \frac{t}{t+d(x,y)}$ for all x, y □ X and all t > 0.Then (X,M, *) is a fuzzy metric space. It is called the fuzzy metric space induced by the metric d.
Lemma 2.1. Let (X,M, *) be a fuzzy metric space .If there exist k □ $(0,1)$ such that $M(x,y,kt) \geq M(x,y,t)$ for all $x, y \in X$ and $t > 0$ then x = y .
Definition 2.3. ([5]) Let (X,M, *) be a fuzzy metric space. A sequence { xn } in X is said to converge to a point x □ X if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all t > 0. Further, the sequence {xn} is said to be a Cauchy sequence if $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$ for all t > 0 and p > 0. The space is said to be complete if every Cauchy sequence in X converges to a point in X.

III. Compatible Maps

In this section, we give the concept of different types of compatible maps and some properties of them for our main result.
Definition 3.1. ([10]) Two maps A and S from a fuzzy metric space (X,M, *) into itself are said to be R weakly commuting if there exists a positive real number R such that for each x □ X M(ASx, SAx,Rt) ≥ M(Ax, Sx, t)
Definition 3.2. ([7]) Two maps A and B from a fuzzy metric space (X,M, *) into itself are said to be compatible if $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ for all t > 0, whenever {xn} is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x for some x □ X.
Definition 3.3. ([1]) Two maps A and B from a fuzzy metric space (X,M, *) into itself are said to be compatible of type (□) If $\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1$ $\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1$ for all t > 0, whenever {xn} is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x for some x □ X.
Definition 3.4. ([2]) Two maps A and B from a fuzzy metric space (X,M, *) into itself are said to be compatible of type (β) if $\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1$ for all t > 0, whenever {xn} is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x for some x □ X.
Definition 3.5. Two maps A and B from a fuzzy metric space (X,M, *) into itself are said to be weak -compatible if they commute at their coincidence points, i.e., Ax = Bx implies ABx = BAx.
Definition 3.6. A pair (A, S) of self-maps of a fuzzy metric space (X,M, *) is said to be semi-compatible if $lim_{n\to\infty} Ax_n = sx$ whenever {xn} is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x * X. It follows that (A, S) is semi-compatible and Ay = Sy then ASy = SAy.

Remark 3.1. Let (A, S) be a pair of self-maps of a fuzzy metric space (X,M, *). Then (A, S) is R-weakly commuting implies that (A, S) is compatible, which implies that (A, S) is weak-compatible. But the converse is not true.

Theorem 3.1. Let A, B, S , T , L and M be self maps on a complete Fuzzy metric space $(X, M, □)$ with $t □ t \geq t$ for all t □ [0,1], satisfying

(a) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

(b) there exist a constant k□(0,1) such that

$$M^2(Lx, My, kt) * [M(ABx, Lx, kt).M(STy, My, kt)] * \left[\frac{1 + M(Lx, My, t)}{2}\right] \geq [pM(ABx, Lx, t) + qM(ABx, STy, t)]M(ABx, My, 2kt)$$

for all $x, y □ X$ and $t > 0$ where $0 < p, q < 1$ such that $p + q = 1$;

(c) AB=BA, ST=TS, LB=BL, MT=TM

(d) either AB or L is continuous;

(e) the pair(Ab,L) is compatible of type (□) and (M,ST) is weak compatible. Then A, B, S, T, L and M have a unique common fixed point.

Proof : Let x_0 be an arbitrary point of X. By (a) there exist $x_1, x_2 \in X$ such that $Lx_0 = STx_1 = y_0$ And $Mx_1 = ABx_1 = y_1$. Inductively ,we can construct sequences$\{x_n\}$ *and* $\{y_n\}$ in X such that $Lx_{2n} = ST_{2n+1} = y_{2n}$ *and* $Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for n = 0, 1, 2 ……

Step 1. By Taking $x = x_{2n}$ and $y = x_{2n+1}$ in (b) we have

$$M^2(Lx_{2n}, Mx_{2n+1}, kt) * [M(ABx_{2n}, Lx_{2n}, kt).M(STx_{2n+1}, Mx_{2n+1}, kt)] * \left[\frac{1 + M(Lx_{2n}, Mx_{2n+1}, kt)}{2}\right]$$
$$\geq [pM(ABx_{2n}, Lx_{2n}, t) + qM(ABx_{2n}, STx_{2n+1}, t)]M(ABx_{2n}, Mx_{2n+1}, 2kt)$$
$$M^2(y_{2n}, y_{2n+1}, kt) * [M(y_{2n-1}, y_{2n}, kt).M(y_{2n}, y_{2n+1}, kt)] * \left[\frac{1 + M(y_{2n}, y_{2n+1}, kt)}{2}\right]$$
$$\geq \left[(p + q)M(y_{2n}, y_{2n-1}, t)\right]M(y_{2n-1}, y_{2n+1}, 2kt)$$
$$M(y_{2n}, y_{2n+1}, kt).[M(y_{2n-1}, y_{2n}, kt) * M(y_{2n}, y_{2n+1}, kt)]$$
$$\geq \left[(p + q)M(y_{2n}, y_{2n-1}, t)\right]M(y_{2n-1}, y_{2n+1}, 2kt)$$
$$M^2(y_{2n}, y_{2n+1}, kt).M(y_{2n-1}, y_{2n+1}, 2kt)$$
$$\geq M(y_{2n-1}, y_{2n}, t)M(y_{2n-1}, y_{2n+1}, 2kt)$$

Hence, we have

$$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t)$$

Similarly we also have

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$$

In general for all n even or odd, we have

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$$

for k □ $(0, 1)$ *and all* $t > 0$. Thus by Lemma (2.1), $\{y_n\}$ is a Cauchy sequence in X. Since (X, M, *) is complet ,it converges to a point z in X.

Also its subsequences converges as follows: $\{Lx_{2n}\} \to z, \{ABx_{2n}\} \to z, \{Mx_{2n+1}\} \to z$, *and* $\{STx_{2n+1}\} \to z$.

Case I. AB is continuous. Since AB is continuous, $AB(AB)x_{2n} \to ABz$ and $L(AB)x_{2n} \to ABz$

SInce(AB, L) is compatible of type □ , $L(AB)x_{2n} \to ABz$

Step 2 . By taking $x = ABx_{2n}$ *and* $y = x_{2n+1}$ *in* (*b*)

$$M^2(LABx_{2n}, Mx_{2n+1}, kt) * [M(ABABx_{2n}, L\,ABx_{2n}, kt).M(STx_{2n+1}, Mx_{2n+1}, kt)] * \left[\frac{1 + M(LABx_{2n}, Mx_{2n+1}, kt)}{2}\right]$$
$$\geq [pM(ABABx_{2n}, LABx_{2n}, t) + qM(ABABx_{2n}, STx_{2n+1}, t)]M(ABABx_{2n}, Mx_{2n+1}, 2kt)$$
$$M^2(ABz, z, kt) * [M(ABz, ABz, kt).M(z, z, kt)] * \left[\frac{1 + M(ABz, z, kt)}{2}\right]$$
$$\geq [pM(ABz, ABz, t) + qM(ABz, z, t)]M(ABz, z, 2kt)$$
$$M^2(ABz, z, kt) \geq [pM(ABz, ABz, t) + qM(ABz, z, t)]M(ABz, z, 2kt)$$
$$M^2(ABz, z, kt) \geq [p + qM(ABz, z, t)]M(ABz, z, kt)$$
$$M(ABz, z, kt) \geq [p + qM(ABz, z, t)]$$
$$\geq [p + qM(ABz, z, t)]$$
$$M(ABz, z, kt) \geq p + qM(ABz, z, kt)$$
$$M(ABz, z, kt) \geq \frac{p}{1 - q} = 1$$

for k □ $(0,1)$ and $t > 0$. Thus ,we have $z = ABz$.

Step 3. By taking $x = z$ *and* $y = x_{2n+1}$ *in* (*b*) we have

$$M^2(Lz, Mx_{2n+1}, kt) □ \left[M(ABz, Lz, kt).M(STx_{2n+1}, Mx_{2n+1}, kt)\right] □ \left[\frac{1 + M(LBz, Mx_{2n+1}, kt)}{2}\right]$$
$$\geq \left[pM(ABz, Lz, t) + qM(ABz, STx_{2n+1}, t)\right]M(ABz, Mx_{2n+1}, 2kt)$$
$$M^2(Lz, z, kt) * [M(z, Lz, kt).M(z, z, kt)] * \left[\frac{1 + M(Lz, z, kt)}{2}\right]$$
$$\geq [pM(z, Lz, t) + qM(z, z, t)]M(z, z, 2kt)$$
$$M^2(Lz, z, kt) * [M(z, Lz, kt).M(z, z, kt)]$$
$$\geq [pM(z, Lz, t) + qM(z, z, t)]M(z, z, 2kt)$$
$$M(Lz, z, kt).[M(Lz, z, kt) * 1] \geq [pM(z, Lz, t) + q]$$
$$M^2(Lz, z, kt) \geq pM(z, Lz, t) + q$$
$$M(Lz, z, kt) \geq pM(z, Lz, t) + q$$
$$M(Lz, z, kt) \geq pM(z, Lz, t) + q$$
$$M(Lz, z, kt) \geq pM(z, Lz, kt) + q$$
$$M(Lz, z, kt) \geq \frac{q}{1 - p} = 1$$

for $k \in (0,1)$ and $t > 0$. Thus , we have $z = Lz = ABz$.

Step 4. By taking $x = Bz$, $y = x_{2n+1}$ in (*b*) we have

$$M^2(LBz, Mx_{2n+1}, kt) □ \left[M(ABBz, LBz, kt)M(STx_{2n+1}, Mx_{2n+1}, kt)\right] □ \left[\frac{1 + M(LBz, Mx_{2n+1}, kt)}{2}\right]$$
$$\geq \left[pM(ABBz, LBz, t) + qM(ABBz, STx_{2n+1}, t)\right]M(ABBz, Mx_{2n+1}, 2kt)$$

Since $AB = BA$ and $BL = LB$, we have $L(Bz) = B(Lz) = Bz$ and $AB(Bz) = B(ABz) = Bz$. Letting $n \to \infty$, we have

$$M^2(Bz, z, kt) * [M(Bz, Bz, kt)M(z, z, kt)] \geq [pM(Bz, Bz, t) + qM(Bz, z, t)]M(Bz, z, 2kt)$$
$$M^2(Bz, z, kt) \geq [p + qM(Bz, z, t)]M(Bz, z, 2kt)]$$
$$M^2(Bz, z, kt) \geq [p + qM(Bz, z, t)]M(Bz, z, kt)]$$
$$M(Bz, z, kt) \geq [p + qM(Bz, z, t)]$$
$$\geq [p + qM(Bz, z, kt)]$$

International Journal of Engineering Innovation & Research
Volume 2, Issue 1, ISSN: 2277 – 5668

$$M(Bz,z,kt) \geq \frac{p}{1-q} = 1$$

For $k \in (0,1)$ and all $t > 0$. Thus we have $z = Bz$. Since $z = ABz$, we have $z = Az$ therefore , $z = Az = Bz = Lz$.

Step 5. Since $L(X) \subset ST(X)$, there exist $v \in X$ such that $z = Lz = STv$, by taking $x = x_{2n}$, $y = v$ in (b), we have

$$M^2(Lx_{2n}, Mv, kt) * [M(ABx_{2n} Lx_{2n}, kt)M(STv, Mv, kt)] * \left[\frac{1 + M(Lx_{2n}, Mv, kt)}{2}\right]$$
$$\geq [pM(ABx_{2n} Lx_{2n}, t) + qM(ABx_{2n} STv, t)]M(ABx_{2n} Mv, 2kt)$$

Which implies that $n \to \infty$

$$M^2(z,Mv,kt) * [M(z,z,kt)M(z,Mv,kt)] * \left[\frac{1 + M(z,Mv,kt)}{2}\right]$$
$$\geq [pM(z,z,t) + qM(z,z,t)]\, M(z,Mv,2kt)$$
$$M^2(z,Mv,kt) * [M(z,z,kt)M(z,Mv,kt)]$$
$$\geq [pM(z,z,t) + qM(z,z,t)]\, M(z,Mv,2kt)$$
$$M^2(z,Mv,kt) * [M(z,Mv,kt)] \geq [p+q]\, M(z,Mv,2kt)$$
$$M^2(z,Mv,kt) \geq M(z,Mv,2kt)$$
$$M(z,Mv,kt) \geq M(z,Mv,2kt)$$
$$M(z,Mv,kt) \geq M(z,Mv,t)$$
$$\geq M(z,Mv,t)$$

Thus by lemma (2.1), we have $z = Mv$ and so $z = Mv$. Since (M, ST) is weak compatible, we have $STMv = MSTv$. Thus $STz = Mz$.

Step 6. By taking $x = x_{2n}$, $y = z$ in (b) and using step (5), we have

$$M^2(Lx_{2n} Mz, kt) * [M(ABx_{2n}, Lx_{2n}, kt)M(STz, Mz, kt)] * \left[1 + \frac{M(Lx_{2n} Mz, kt)}{2}\right]$$
$$\geq [pM(ABx_{2n} Lx_{2n} t) + qM(ABx_{2n}, STz, t)]M(ABx_{2n} Mz, 2kt)$$

Which implies that as $n \to \infty$

$$M^2(z,Mz,kt) * [M(z,z,kt).(Mz,Mz,kt)] * \left[1 + \frac{M(z,Mz,kt)}{2}\right]$$
$$\geq [pM(z,z,t) + qM(z,Mz,t)]M(z,Mz,2kt)$$
$$M^2(z,Mz,kt) * [M(z,z,kt).M(Mz,Mz,kt)]$$
$$\geq [p + qM(z,Mz,t)]M(z,Mz,2kt)$$
$$M^2(z,Mz,kt) \geq [p + qM(z,Mz,t)]M(z,Mz,2kt)$$
$$M^2(z,Mz,kt) \geq [p + qM(z,Mz,t)]M(z,Mz,kt)$$
$$M(z,Mz,kt) \geq [p + qM(z,Mz,t)]M(z,Mz,kt)$$
$$\geq [p + qM(z,Mz,t)]$$
$$M(z,Mz,kt) \geq p + qM(z,Mz,kt)$$
$$M(z,Mz,kt) \geq \frac{p}{1-q} = 1.$$

Thus we have $z = Mz$ and therefore $z = Az = Mz = Bz = Lz = STz$.

Step 7. By taking $x = x_{2n}$, $y = Tz$ in (b), we have

$$M^2(Lx_{2n} MTz, kt) * [M(ABx_{2n}, Lx_{2n}, kt)M(STTz, MTz, kt)] * \left[1 + \frac{M(Lx_{2n} MTz, kt)}{2}\right]$$
$$\geq [pM(ABx_{2n} Lx_{2n} t) + qM(ABx_{2n} STTz, t)]M(ABx_{2n} MTz, 2kt)$$

Since $MT = TM$ and $TS = ST$, we have $MTz = TMz = Tz$ and $ST(Tz) = T(STz) = Tz$. Letting $n \to \infty$we have

$$M^2(z,Tz,kt) * [M(z,z,kt)M(Tz,Tz,kt)] * \left[1 + \frac{M(z,Tz,kt)}{2}\right]$$
$$\geq [pM(z,z,t) + qM(z,Tz,t)]M(z,Tz,2kt)$$
$$M^2(z,Tz,kt) * [M(z,z,kt)M(Tz,Tz,kt)]$$
$$\geq [pM(z,z,t) + qM(z,Tz,t)]M(z,Tz,2kt)$$
$$M^2(z,Tz,kt) \geq [p + qM(z,Tz,t)]M(z,Tz,kt)$$
$$M(z,Tz,kt) \geq [p + qM(z,Tz,t)]$$
$$\geq [p + qM(z,Tz,kt)]$$
$$M(z,Tz,kt) \geq \frac{p}{1-q} = 1$$

Thus we have $z = Tz$. Since $Tz = STz$, we also have $z = Sz$. Therefore $z = Az = Bz = Lz = Mz = Sz = Tz$, that is z is he common fixed point of the six maps.

Case II. L is continuous.

Since L is continuous, $LLx_{2n} \to Lz$ and $L(AB)x_{2n} \to Lz$. Since (AB, L) is compatible of type α, therefore $(AB)Lx_{2n} = Lz$.

Step 8. By taking $x = Lx_{2n}, y = x_{2n+1}$ in (b) we have

$$M^2(z,Lz,kt) * [M(Lz,Lz,kt)M(z,z,kt)] * \left[1 + \frac{M(z,Lz,kt)}{2}\right]$$
$$\geq [pM(Lz,Lz,t) + qM(z,Lz,t)]M(z,Lz,2kt)$$
$$M^2(z,Lz,kt) * [M(Lz,Lz,kt)M(z,z,kt)]$$
$$\geq [pM(Lz,Lz,t) + qM(z,Lz,t)]M(z,Lz,2kt)$$
$$M^2(z,Lz,kt) \geq [p + qM(z,Lz,t)]M(z,Lz,2kt)$$
$$M(z,Lz,kt) \geq [p + qM(z,Lz,t)]$$
$$\geq [p + qM(z,Lz,kt)]$$
$$M(z,Lz,kt) \geq \frac{p}{1-q} = 1$$

Thus we have $z = Lz$ and using step 5-7,we have $z = Lz = Mz = Sz = Tz$.

Step 9. Since $M(X) \subseteq AB(X)$, there exist $v \in X$ such that $z = Mz = ABv$. By taking $x = v$, $y = x_{2n+1}$ in(b),we have

$$M^2(Lv, Mx_{2n+1}, kt) * [M(ABv, Lv, kt)M(STx_{2n+1}, Mx_{2n+1}, kt)] * \left[1 + \frac{M(Lv, Mx_{2n+1}, kt)}{2}\right]$$
$$\geq [pM(ABv, Lv, t) + qM(ABv, STx_{2n+1}t)]M(ABv, Mx_{2n+1}, 2kt)$$

$$M^2(z,Lv,kt) * [M(z,Lv,kt)M(z,z,kt)] * \left[1 + \frac{M(Lv,z,kt)}{2}\right]$$
$$\geq [pM(z,Lv,t) + qM(z,z,t)]M(z,z,2kt)$$
$$M^2(z,Lv,kt) * [M(z,Lv,kt)M(z,z,kt)]$$
$$\geq [pM(z,Lv,t) + qM(z,z,t)]M(z,z,2kt)$$
$$M^2(z,Lv,kt) * [M(z,Lv,kt)] \geq [pM(z,Lv,t) + q]$$
$$\geq pM(z,Lv,t) + q$$
$$M(z,Lv,kt) \geq pM(z,Lv,kt) + q$$
$$M(z,Lv,kt) \geq \frac{q}{1-p} = 1$$

Thus we have $z = Lv = ABv$. Since(AB,L) is compatible of type α ,we have $Lz = ABz$ and using step 4 we also $z = Bz$. Therefore $z = Az = Bz = Sz = Tz = Lz = Mz$, that is z is the common fixed point ot the six maps in this case also.

Step 10. For uniqueness, let w(w≠z) be another common fixed point of A, B, S, T, L, M. Taking x = z , y= w in (b) we have,

$$M^2(Mw,Lz,kt) * [M(ABz,Lz,kt)M(STw,Mw,kt)] * \left[1+\frac{M(Mw,Lz,kt)}{2}\right] \geq [pM(ABz,Lz,t)+qM(ABz,STw,t)]M(ABz,Mw,2kt)$$

$$M^2(w,z,kt) * [M(ABz,Lz,kt)M(STw,Mw,kt)] * \left[1+\frac{M(w,z,kt)}{2}\right] \geq [pM(ABz,Lz,t)+qM(ABz,STw,t)]M(ABz,Mw,2kt)$$

$$M^2(w,z,kt) * [M(ABz,Lz,kt)M(STw,Mw,kt)] \geq [pM(ABz,Lz,t)+qM(ABz,STw,t)]M(ABz,Mw,2kt)$$

Which implies that

$$M^2(w,z,kt) \geq [p+qM(z,w,t)]M(z,w,2kt)$$
$$\geq [p+qM(z,w,t)]M(z,w,kt)$$
$$M(w,z,kt) \geq [p+qM(z,w,t)]$$
$$M(w,z,kt) \geq p+qM(z,w,kt)$$
$$M(w,z,kt) \geq \frac{p}{1-q} = 1$$

Thus we have z = w. This completes the proof of the theorem.

Corollary 3.2. Let A, B, S , T , L and M be self maps on a complete Fuzzy metric space $(X, M, *)$ with $t * t \geq t$ for all $t \in [0,1]$, satisfying

(a) L(X)⊂S(X) ,M(X)⊂A(X);

(b) there exist a constant k∈(0,1) such that

$$M^2(Lx,My,kt) * [M(Ax,Lx,kt).M(Sy,My,kt)] * \left[\frac{1+M(Lx,My,t)}{2}\right] \geq [pM(Ax,Lx,t)+qM(Ax,Sy,t)]M(ABx,My,2kt)$$

for all x, y∈X and t>0 where 0<p, q<1 such that p+q= 1;

(c) either A or L is continuous;

(d) the pair (L, A) is compatible of type (β) and (M,S) is weak compatible then A, S, L and M have a unique common fixed point.

References

[1] Y.J. Cho, Fixed point in fuzzy metric space, J. Fuzzy Math. 5 (1997), 949–962.

[2] Y.J. Cho, H.K. Pathak, S.M. Kang and J.S. Jung, Fuzzy Sets and System 93 (1998), 99–111.

[3] Y.J. Cho, B.K. Sharma and D.R. Sahu, Semi-compatibility and fixed points, Math Japonica 42 (1995), 91–98.

[4] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and System 64 (1994), 395–399.

[5] M. Grebiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and System 27 (1988), 385–389.

[6] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975), 326–334.

[7] S.N. Mishra, N. Mishra, S.L. Singh, Common fixed point of maps in fuzzy metric space, Int. J. Math. Math. Sci. 17 (1994), 253–258.

[8] B. Singh and M.S. Chauhan, Common fixed point of compatible maps in fuzzy metric space, Fuzzy Sets and System 115 (2000), 471–475.

[9] R. Vasuki, Common fixed point theorem in a fuzzy metric space, Fuzzy Sets and System 97 (1998), 395–397.

[10] R. Vasuki, Common fixed points for R-weakly commuting maps in fuzzy metric space, Indian J. Pure Appl. Math. 30 (1999), 419–423.

[11] L.A. Zadeh, Fuzzy sets, Inform and Control 89 (1965), 338–353.

[12] 12 .I. Kramosil and J.Michalek, Fuzzy metrics and Statistical metric spaces ,Kybernetika(Prague) 11(5) (1975) 336-344. MR0410633(53≠ 14381)

[13] R. Vasuki "Common fixed points for R-weakly commuting maps in fuzzy metric spaces ",Indian J. Pure Appl. Bull. Math. 30 (1999), 419-423.

[14] R. P. Pant "Common fixed points of four mappings, " Bull. Cal. Math. Soc .90 (1998).251-258.

[15] P. Balasubramaniam, S. Muralisankar ,R.P. Pant ,"Common fixed points of four mappings in a fuzzy metric spaces ".J. Fuzzy Math. 10(2) (2002),379-384.